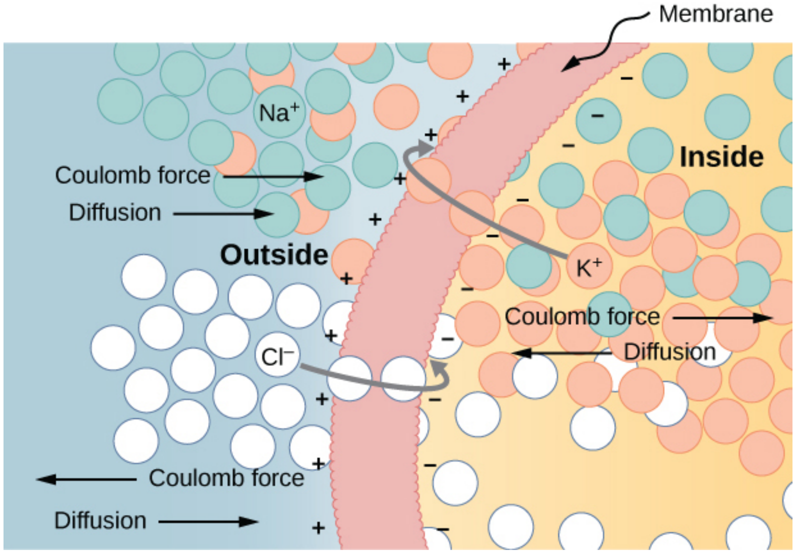


Figure 8.10 The semipermeable membrane of a biological cell has different concentrations of ions on its interior surface than on its exterior. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. In this way, the exterior of the membrane acquires a positive charge and its interior surface acquires a negative charge, creating a potential difference across the membrane. The membrane is normally impermeable to Na+ (sodium ions).

Visit the **PhET Explorations: Capacitor Lab (https://openstaxcollege.org/l/21phetcapacitor)** to explore how a capacitor works. Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electrical field in the capacitor. Measure the voltage and the electrical field.

8.2 | Capacitors in Series and in Parallel

Learning Objectives

By the end of this section, you will be able to:

- Explain how to determine the equivalent capacitance of capacitors in series and in parallel combinations
- Compute the potential difference across the plates and the charge on the plates for a capacitor in a network and determine the net capacitance of a network of capacitors

Several capacitors can be connected together to be used in a variety of applications. Multiple connections of capacitors behave as a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. Capacitors can be arranged in two simple and common types of connections, known as *series* and *parallel*, for which we can easily calculate the total capacitance. These two basic combinations, series and parallel, can also be used as part of more complex connections.

The Series Combination of Capacitors

Figure 8.11 illustrates a series combination of three capacitors, arranged in a row within the circuit. As for any capacitor, the capacitance of the combination is related to the charge and voltage by using **Equation 8.1**. When this series combination is connected to a battery with voltage *V*, each of the capacitors acquires an identical charge *Q*. To explain, first note that the charge on the plate connected to the positive terminal of the battery is $+Q$ and the charge on the plate connected to the negative terminal is $-Q$. Charges are then induced on the other plates so that the sum of the charges on all plates, and the sum of charges on any pair of capacitor plates, is zero. However, the potential drop $V_1 = Q/C_1$ on one capacitor may be different from the potential drop $V_2 = Q/C_2$ on another capacitor, because, generally, the capacitors may have different capacitances. The series combination of two or three capacitors resembles a single capacitor with a smaller capacitance. Generally, any number of capacitors connected in series is equivalent to one capacitor whose capacitance (called the *equivalent capacitance*) is smaller than the smallest of the capacitances in the series combination. Charge on this equivalent capacitor is the same as the charge on any capacitor in a series combination: That is, *all capacitors of a series combination have the same charge*. This occurs due to the conservation of charge in the circuit. When a charge *Q* in a series circuit is removed from a plate of the first capacitor (which we denote as $-Q$), it must be placed on a plate of the second capacitor (which we denote as $+Q$), and so on.

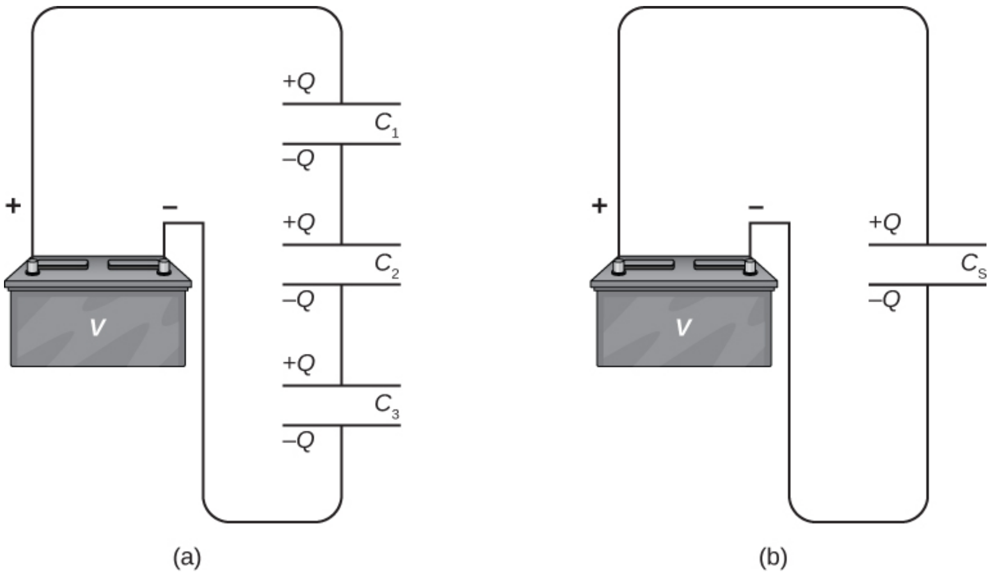


Figure 8.11 (a) Three capacitors are connected in series. The magnitude of the charge on each plate is *Q*. (b) The network of capacitors in (a) is equivalent to one capacitor that has a smaller capacitance than any of the individual capacitances in (a), and the charge on its plates is *Q*.

We can find an expression for the total (equivalent) capacitance by considering the voltages across the individual capacitors. The potentials across capacitors 1, 2, and 3 are, respectively, $V_1 = Q/C_1$, $V_2 = Q/C_2$, and $V_3 = Q/C_3$. These potentials must sum up to the voltage of the battery, giving the following potential balance:

$$V = V_1 + V_2 + V_3.$$

Potential *V* is measured across an equivalent capacitor that holds charge *Q* and has an equivalent capacitance C_S. Entering the expressions for V_1, V_2, and V_3, we get

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Canceling the charge *Q*, we obtain an expression containing the equivalent capacitance, C_S, of three capacitors connected in series:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

This expression can be generalized to any number of capacitors in a series network.

Series Combination

For capacitors connected in a **series combination**, the reciprocal of the equivalent capacitance is the sum of reciprocals of individual capacitances:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots. \tag{8.7}$$

Example 8.4

Equivalent Capacitance of a Series Network

Find the total capacitance for three capacitors connected in series, given their individual capacitances are $1.000\ \mu\text{F}$, $5.000\ \mu\text{F}$, and $8.000\ \mu\text{F}$.

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using **Equation 8.7** with three terms.

Solution

We enter the given capacitances into **Equation 8.7**:

$$\begin{aligned}\frac{1}{C_S} &= \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\\ &= \frac{1}{1.000\ \mu\text{F}}+\frac{1}{5.000\ \mu\text{F}}+\frac{1}{8.000\ \mu\text{F}}\\ \frac{1}{C_S} &= \frac{1.325}{\mu\text{F}}.\end{aligned}$$

Now we invert this result and obtain $C_S = \frac{\mu\text{F}}{1.325} = 0.755\ \mu\text{F}.$

Significance

Note that in a series network of capacitors, the equivalent capacitance is always less than the smallest individual capacitance in the network.

The Parallel Combination of Capacitors

A parallel combination of three capacitors, with one plate of each capacitor connected to one side of the circuit and the other plate connected to the other side, is illustrated in **Figure 8.12**(a). Since the capacitors are connected in parallel, *they all have the same voltage V across their plates*. However, each capacitor in the parallel network may store a different charge. To find the equivalent capacitance C_P of the parallel network, we note that the total charge Q stored by the network is the sum of all the individual charges:

$$Q = Q_1 + Q_2 + Q_3.$$

On the left-hand side of this equation, we use the relation $Q = C_P V$, which holds for the entire network. On the right-hand side of the equation, we use the relations $Q_1 = C_1 V$, $Q_2 = C_2 V$, and $Q_3 = C_3 V$ for the three capacitors in the network. In this way we obtain

$$C_P V = C_1 V + C_2 V + C_3 V.$$

This equation, when simplified, is the expression for the equivalent capacitance of the parallel network of three capacitors:

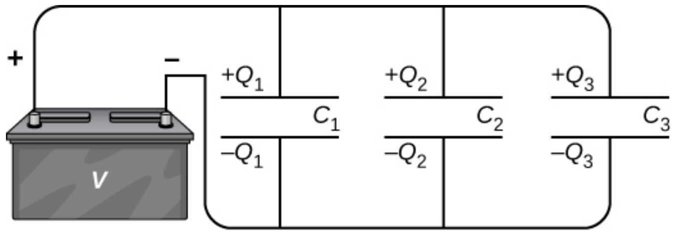
$$C_P = C_1 + C_2 + C_3.$$

This expression is easily generalized to any number of capacitors connected in parallel in the network.

Parallel Combination

For capacitors connected in a **parallel combination**, the equivalent (net) capacitance is the sum of all individual capacitances in the network,

$$C_P = C_1 + C_2 + C_3 + \cdots. \tag{8.8}$$



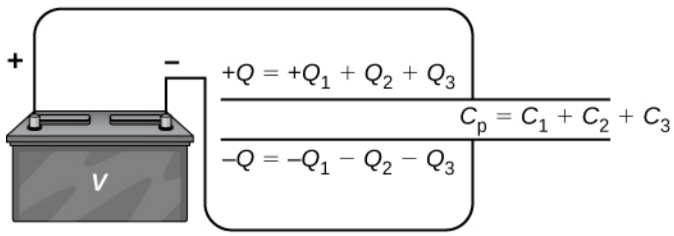


Figure 8.12 (a) Three capacitors are connected in parallel. Each capacitor is connected directly to the battery. (b) The charge on the equivalent capacitor is the sum of the charges on the individual capacitors.

Example 8.5

Equivalent Capacitance of a Parallel Network

Find the net capacitance for three capacitors connected in parallel, given their individual capacitances are $1.0\,\mu\text{F},\ 5.0\,\mu\text{F},\ \text{and}\ 8.0\,\mu\text{F}.$

Strategy

Because there are only three capacitors in this network, we can find the equivalent capacitance by using **Equation 8.8** with three terms.

Solution

Entering the given capacitances into **Equation 8.8** yields

$$\begin{aligned} C_\text{P} &= C_1 + C_2 + C_3 = 1.0\,\mu\text{F} + 5.0\,\mu\text{F} + 8.0\,\mu\text{F} \\ C_\text{P} &= 14.0\,\mu\text{F}. \end{aligned}$$

Significance

Note that in a parallel network of capacitors, the equivalent capacitance is always larger than any of the individual capacitances in the network.

Capacitor networks are usually some combination of series and parallel connections, as shown in **Figure 8.13**. To find the net capacitance of such combinations, we identify parts that contain only series or only parallel connections, and find their equivalent capacitances. We repeat this process until we can determine the equivalent capacitance of the entire network. The following example illustrates this process.

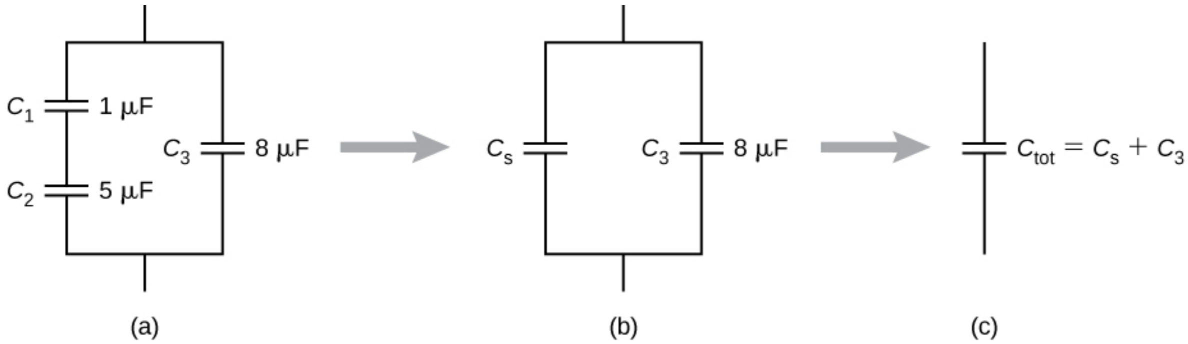


Figure 8.13 (a) This circuit contains both series and parallel connections of capacitors. (b) C_1 and C_2 are in series; their equivalent capacitance is C_S. (c) The equivalent capacitance C_S is connected in parallel with C_3. Thus, the equivalent capacitance of the entire network is the sum of C_S and C_3.

Example 8.6

Equivalent Capacitance of a Network

Find the total capacitance of the combination of capacitors shown in **Figure 8.13**. Assume the capacitances are known to three decimal places $(C_1 = 1.000\ \mu\text{F},\ C_2 = 5.000\ \mu\text{F},\ C_3 = 8.000\ \mu\text{F})$. Round your answer to three decimal places.

Strategy

We first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S, is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their equivalent capacitance C_S is obtained with **Equation 8.7**:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000\ \mu\text{F}} + \frac{1}{5.000\ \mu\text{F}} = \frac{1.200}{\mu\text{F}} \Rightarrow C_S = 0.833\ \mu\text{F}.$$

Capacitance C_S is connected in parallel with the third capacitance C_3, so we use **Equation 8.8** to find the equivalent capacitance C of the entire network:

$$C = C_S + C_3 = 0.833\ \mu\text{F} + 8.000\ \mu\text{F} = 8.833\ \mu\text{F}.$$

Example 8.7

Network of Capacitors

Determine the net capacitance C of the capacitor combination shown in **Figure 8.14** when the capacitances are $C_1 = 12.0\ \mu\text{F},\ C_2 = 2.0\ \mu\text{F},$ and $C_3 = 4.0\ \mu\text{F}$. When a 12.0-V potential difference is maintained across the combination, find the charge and the voltage across each capacitor.

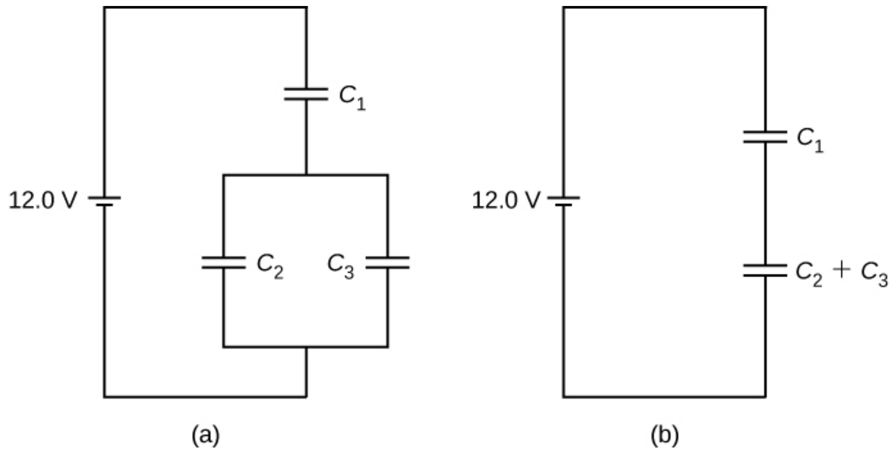


Figure 8.14 (a) A capacitor combination. (b) An equivalent two-capacitor combination.

Strategy

We first compute the net capacitance C_{23} of the parallel connection C_2 and C_3. Then C is the net capacitance of the series connection C_1 and C_{23}. We use the relation $C = Q/V$ to find the charges Q_1, Q_2, and Q_3, and the voltages V_1, V_2, and V_3, across capacitors 1, 2, and 3, respectively.

Solution

The equivalent capacitance for C_2 and C_3 is

$$C_{23} = C_2 + C_3 = 2.0\,\mu\text{F} + 4.0\,\mu\text{F} = 6.0\,\mu\text{F}.$$

The entire three-capacitor combination is equivalent to two capacitors in series,

$$\frac{1}{C} = \frac{1}{12.0\,\mu\text{F}} + \frac{1}{6.0\,\mu\text{F}} = \frac{1}{4.0\,\mu\text{F}} \Rightarrow C = 4.0\,\mu\text{F}.$$

Consider the equivalent two-capacitor combination in **Figure 8.14**(b). Since the capacitors are in series, they have the same charge, $Q_1 = Q_{23}$. Also, the capacitors share the 12.0-V potential difference, so

$$12.0\text{ V} = V_1 + V_{23} = \frac{Q_1}{C_1} + \frac{Q_{23}}{C_{23}} = \frac{Q_1}{12.0\,\mu\text{F}} + \frac{Q_1}{6.0\,\mu\text{F}} \Rightarrow Q_1 = 48.0\,\mu\text{C}.$$

Now the potential difference across capacitor 1 is

$$V_1 = \frac{Q_1}{C_1} = \frac{48.0\,\mu\text{C}}{12.0\,\mu\text{F}} = 4.0\text{ V}.$$

Because capacitors 2 and 3 are connected in parallel, they are at the same potential difference:

$$V_2 = V_3 = 12.0\text{ V} - 4.0\text{ V} = 8.0\text{ V}.$$

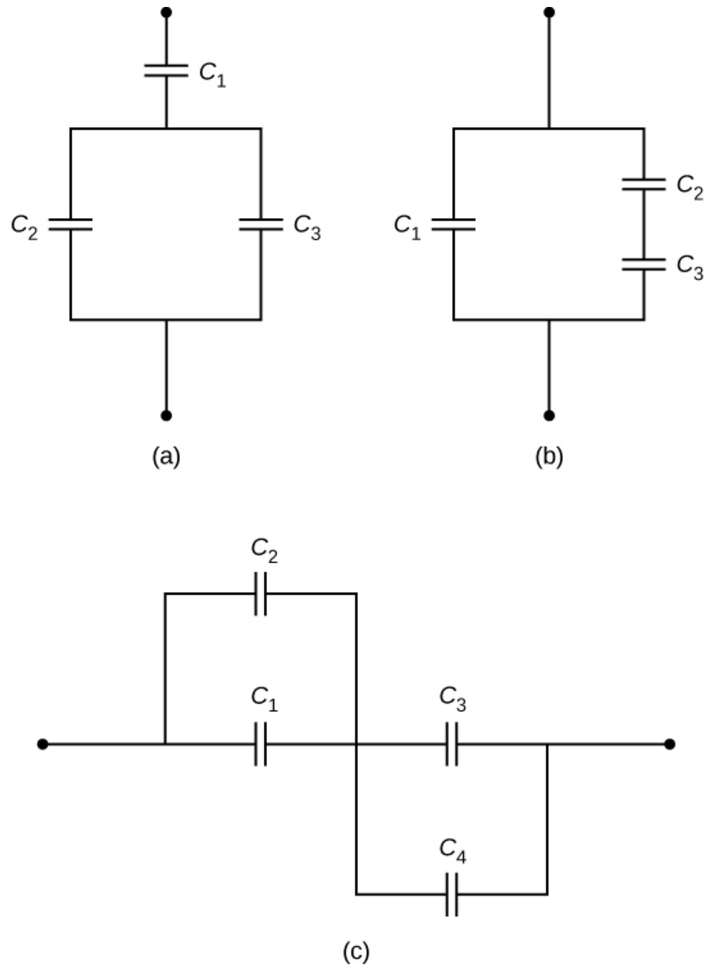
Hence, the charges on these two capacitors are, respectively,

$$Q_2 = C_2 V_2 = (2.0\,\mu\text{F})(8.0\text{ V}) = 16.0\,\mu\text{C},$$
$$Q_3 = C_3 V_3 = (4.0\,\mu\text{F})(8.0\text{ V}) = 32.0\,\mu\text{C}.$$

Significance

As expected, the net charge on the parallel combination of C_2 and C_3 is $Q_{23} = Q_2 + Q_3 = 48.0\,\mu\text{C}$.

8.5 Check Your Understanding Determine the net capacitance C of each network of capacitors shown below. Assume that $C_1 = 1.0\,\text{pF}$, $C_2 = 2.0\,\text{pF}$, $C_3 = 4.0\,\text{pF}$, and $C_4 = 5.0\,\text{pF}$. Find the charge on each capacitor, assuming there is a potential difference of 12.0 V across each network.



8.3 | Energy Stored in a Capacitor

Learning Objectives

By the end of this section, you will be able to:

- Explain how energy is stored in a capacitor
- Use energy relations to determine the energy stored in a capacitor network

Most of us have seen dramatizations of medical personnel using a defibrillator to pass an electrical current through a patient's heart to get it to beat normally. Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics to supply energy when batteries are charged (**Figure 8.15**). Capacitors are also used to supply energy for flash lamps on cameras.